Spring 2002

Ph.D. Qualifying Examination
Reactor Theory and Experimentation

1. (15 min.) Consider one-group diffusion theory for a reactor that is critical, cubical, homogeneous, and surrounded by vacuum. The reactor has width w, and you should take the origin of the (x,y,z) coordinate system to be at the reactor center.

   a) (60%) What is the scalar-flux distribution, $\phi(x,y,z)$, in this reactor? Define any parameters that appear in your answer in terms of the reactor width and/or the one-group cross sections.
   b) (40%) Given that the reactor is critical, what is an equation that relates material properties (one-group cross sections) to the reactor width?

2. (25 min.) Consider neutrons slowing down from 1 MeV to thermal energies in an infinite homogeneous medium of some isotope that is **not hydrogen**. Assume that in this "slowing-down" energy range, there is no absorption, scattering is s-wave only, and the scattering cross section is "constant" with energy. For this problem, in this energy range, the energy-dependent scalar flux is approximately a constant times a very simple function of energy: $\phi(E) = C\,f(E)$, where C is a constant.

   a) (20%) What is that simple function of energy, denoted $f(E)$ above?
   b) (20%) The slowing-down equation in the energy range of interest in this problem is:

   $$\Sigma_s \phi(E) = \int_E^{E/\alpha} dE' \frac{\Sigma_s}{(1-\alpha)E'} \phi(E').$$

   Insert your answer from part (a) into this equation and show that it is a solution.
   c) (5 min.) What is the parameter α in the equation above? Your answer should be in terms of a property or properties of the isotope that constitutes the infinite medium.
   d) (10 min.) The slowing-down density, $q(E)$, is the number of neutrons per cm^3 per second that slow down from energies above E to energies below E. In this steady-state problem, in the slowing-down energy range, $q(E)$ must equal S, the source-rate density at which high-energy neutrons are being born in the medium.

   Find the constant C in the expression for the scalar flux, in terms of S and properties of the isotope that constitutes the infinite medium. Use the GENERAL definition of $q(E)$ [hint: this involves a double integral] and the knowledge that $q = S$.

3. (30 min.) The point reactor kinetic equations for one effective delayed-neutron group can be written, in more-or-less standard notation, as

$$\frac{dP}{dt} = \left[\frac{\rho_0 - \beta}{\Lambda}\right] P(t) + \lambda C(t),$$
$$\frac{dC}{dt} = \frac{\beta}{\Lambda} P(t) - \lambda C(t).$$

(a) (30%) Explain the meaning of each of the symbols appearing in these point kinetic equations.

(b) (25%) Consider the case of a (fast) reactor, with parameters $\beta = 10^{-2}$, $\Lambda = 10^{-7}$ secs, and $\lambda = 10^{-1}$ secs. Further, suppose the reactor initially is at steady state, with $P(0) = 300$, but a reactivity insertion of $\rho = 10^{-4}$ occurs at *t=0.*

(20%) Given that the general solution (after *t=0*) of the above point kinetic equations for the power is

$$P(t) = C_1 \exp(1.01010 \times 10^{-3}\, t) + C_2 \exp(-.99001 \times 10^{5}\, t),$$

for arbitrary constants C_1 and C_2, show that $C_1 = 303.030$ and $C_2 = -3.030$, for the reactivity insertion described above.

(5%) Shown that the corresponding stable reactor period is on the order of 20 minutes.

(c) (25%) Compute the approximation to *P(0.01)* that results from applying Euler's method, with a single time step of $\Delta t=0.01$ secs, to the system described in the two preceding problem parts. (Euler's method is $y(t+\Delta t)=y(t)+\Delta t * f(t,y(t))$, as applied to a first-order system $\frac{dy}{dt} = f(t,y)$ of ordinary differential equations.)

(d) (20%) Explain **why** Euler's method is highly inaccurate in the preceding application, even though the step size is small compared to the stable period, and (briefly) describe alternative approaches, both numerical and otherwise, that would give a much more accurate approximation to the analytic result.

4. (10 min.) (60%) Write a general form of the neutron transport equation.

(40%) This equation was developed on the basis of several physical assumptions. Describe (at least) two assumptions that impose limitations on its applicability.

5. (20 min.) A fluid fuel reactor consists of uranium dissolved in a molten salt. The mixture achieves criticality by passing through a reactor vessel which contains graphite moderator. The vessel also is surrounded by a graphite reflector.

(a.) Make a sketch of the primary loop of this reactor showing the various components and indicating their functions.
(b.) What is the optimum shape for the reactor vessel and the location and design of the moderator elements in it?
(c.) What design features are needed to assure that criticality does not occur in other parts of the primary loop?

6. (20 min.) The fuel assemblies in a LWR consist of pins with annular fuel pellets consisting of low-enriched uranium dioxide and zircaloy clad. The central void in the pellets is filled with helium gas.

Sketch the fast neutron flux and the thermal neutron flux across a fuel pin and into the water. Explain the shape of the flux for each case.